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Review Article

OSCILLATION OF SECOND ORDER OF NEUTRAL DYNAMIC EQUATION WITH DISTRIBUTED DEVIATING ARGUMENTS

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ABSTRACT

In this paper, we establish some oscillation criteria for second order neutral dynamic equation with deviating arguments of the form:

$$\left(r(t)\left(\left(y)t\right)+p(t)y\left(\tau(t)\right)\right)^{\Delta}\right)^{\Delta}+\int_a^b f(t,y(\delta(t,\xi)))\Delta\xi=0$$

On an arbitrary time scale T. An example illustrating the main result is included.

INTRODUCTION

In a neutral dynamic equation with deviating arguments, the highest order derivative of the unknown function appears with and without deviating arguments. These equations find numerous applications in natural sciences and technology.

In this paper, we study the oscillatory behaviour of second order neutral dynamic equation with distributed deviating arguments of the form

$$\left(r(t)\left(\left(y)t\right)+p(t)y\left(\tau(t)\right)\right)^{\Delta}\right)^{\Delta} \quad (1)$$

$$+\int_a^b f(t,y(\delta(t,\xi)))\Delta\xi=0$$

where $0<a<b$, $\tau(t)$: $\mathbb{T}\to\mathbb{T}$ is right dense continuous function such that $\tau(t)\le t$ and

$\tau(t)\to\infty$ as $t\to\infty$, $\delta(t,\xi)$: $\mathbb{T}\times[a,b]\to\mathbb{T}$ is right dense continuous function decreasing with respect to ξ, $\delta(t,\xi)\le t$ for $\xi\in[a,b]$, $\delta(t,\xi)\to\infty$, as $r(t)>0$ and $0\le p(t)<1$ are real valued right dense continuous function defined on $\mathbb{T}$, $p(t)$ is increasing and (H_1): $\int_{t_0}^{\infty}\frac{1}{r(t)}\Delta t=\infty$,

(H_2): $f:T\times R\to R$ is continuous function such that $uf(t,u)>0$ for all $u\ne 0$ and there exists a positive function $q(t)$ defined on $\mathcal{T}$ such that $\left|f(t,u)\right|\ge q(t)|u|$.

A non trivial function $y(t)$ is said to be a solution of (1) if

$y(t)+p(t)y\left[\tau(t)\right]\in C'_{rd}\left[t_y,\infty\right]$ and

$r(t)\left(y(t)+p(t)y\left(\tau(t)\right)\right)^{\Delta}\in C'_{rd}\left[t_y,\infty\right]$

for $t_y\ge t_0$ and $y(t)$ satisfies equation (1) for $t_y\ge t_0$.

A non trivial solution of Equation (1) is called oscillatory if it is neither eventually positive nor eventually negative, otherwise it is called non oscillatory (Bohner and Peterson, 2001; Bohner and Peterson, 2003; Bohner and Saker, 2004; Akin, et al., 2007).

We note that if $T=R$ we have $\sigma(t)=t$, $\mu(t)=0$ then equation (1) becomes second order neutral differential equation

$$\left(r(t)\left(\left(y)t\right)+p(t)y\left(\tau(t)\right)\right)'\right)'$$

$$+\int_a^b f(t,y(\delta(t,\xi)))d\xi=0$$

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If $T = N$ we have $\sigma(t) = t+1,\ \mu(t) = 1$

then equation (1) becomes

$$\Delta(r(t)(\Delta(y(t)+p(t)y(\tau(t))))) + \sum_{\xi=a}^{b-1} f(t, y(\delta(t,\xi))) = 0$$

For more papers related to neutral dynamic equations with distributed deviating arguments, we refer the reader to [8,9]. The books [1,2] gives time scale calculus and its applications.

MAIN RESULTS

Now we state and prove our main result.

Theorem 4.1

Assume that (H_1) and (H_2) hold. In addition, assume that $r^{\Delta}(t) \geq 0$. Then every solution of Equation (1) oscillates if the inequality

$$\left(r\left(x^{\Delta}\right)^{\Delta}\right)(t) + A(t) r\left(x^{\Delta}\right)\left(\delta(t,a)\right) \leq 0 \quad (2)$$

where $A(t) = \dfrac{(b-a)q(t)(1-p(\delta(t,a))}{r(\delta(t,a))}\left(\dfrac{\delta(t,b)}{2}\right)$

and $x(t) = y(t) + p(t)y(\tau(t))$ (3)

has no eventually positive solution.

Proof

Let $y(t)$ be a non-oscillatory solution of Equation (1). Without loss of generality assume that $y(t) > 0$ for $t \geq t_0$, then $y(\tau(t)) > 0$

and $y\left(\delta(t,\xi)\right) > 0$ for $t \geq t_1 > t_0$ and $b \geq \xi \geq a$.

From Equation (1) and (H_2), we have

$$\left(r(x^{\Delta})\right)^{\Delta}(t) + \int_a^b q(t) y(\delta(t,\xi))\Delta\xi \leq 0 \text{ for all } t \geq t_1 \quad (4)$$

and $(r(x^{\Delta}))(t)$ is an eventually decreasing function.

Now we claim that $r(x^{\Delta}) > 0$ eventually.

If not, there exists a $t_2 \geq t_1$ such that $r(x^{\Delta}(t_2)) = c < 0$,

then we have

$r(t)x^{\Delta}(t) \leq c$ for $t \geq t_2$

and it follows that $x^{\Delta}(t) \leq \dfrac{c}{r(t)}$ (5)

Now integrating Equation (5) from t_2 to t and using (H_1), we obtain $x(t) \leq x(t_2) + c\displaystyle\int_{t_2}^{t} \frac{1}{r(s)}\Delta s \to -\infty$ as $t \to \infty$, which contradicts the fact that $x(t) > 0$ for all $t \geq t_0$. Hence $r\left(x^{\Delta}(t)\right)$ is positive.

Therefore there is a $t_2 \geq t_1$ such that $x^{\Delta}(t) > 0$,

$$x^{\Delta}(t) > 0,\ (r(x^{\Delta}))(t) > 0,\ (r(x^{\Delta}))^{\Delta}(t) < 0,\ t \geq t_2 \quad (6)$$

Now $x(t) = y(t) + p(t)y(\tau(t))$

$y(t) = x(t) - p(t)y(\tau(t))$

$= x(t) - p(t)[x(\tau(t)) - p(\tau(t))\, y(\tau(t))]$

$\geq x(t) - p(t)\, x(\tau(t))$

$\geq (1-p(t))\, x(t)$

Therefore $y(\delta(t,\xi)) \geq (1-p(\delta(t,\xi)))x(\delta(t,\xi))$, (7)

$t \geq t_4 \geq t_3, \xi \in [a,b]$

Multiplying Equation (7) by $q(t)$, then

$$q(t)y(\delta(t,\xi \quad \geq q(t)(1-p(\delta(t,\xi)))x(\delta(t,\xi)),$$

$t \geq t_4 \geq t_3, \xi \in [a,b]$ (8)

Integrating Equation (8) from a to b, we get

$$\int_a^b q(t)y(\delta(t,\xi))\Delta\xi \geq \int_a^b q(t)(1-p(\delta(t,\xi)))x(\delta(t,\xi))\Delta\xi \quad (9)$$

Substituting (9) in (4) we obtain,

$$(r(x^{\Delta}))^{\Delta}(t) + \int_a^b q(t)(1-p(\delta(t,\xi)))x(\delta(t.\xi))\Delta\xi \leq 0 \quad (10)$$

Since $x^{\Delta\Delta}(t) \leq 0$ for some $t \geq t_4$,

$$x(t) = x(t_4) + \int_{t_4}^{t} x^{\Delta}(s)\Delta s \geq (t-t_4)x^{\Delta}(t) \geq \frac{t}{2}x^{\Delta}(t), \quad t \geq t_5 \geq 2t_4$$

Therefore

$$x(\delta(t,\xi)) \geq \left(\frac{\delta(t,b)}{2}\right)x^{\Delta}(\delta(t,\xi)),$$

$$\geq \left(\frac{\delta(t,b)}{2}\right)x^{\Delta}(\delta(t,\xi)),$$

$t \geq t_6 \geq t_5, \xi \in [a,b]$

Substituting the last inequality in Equation (10), we get

$$\left(r(x^{\Delta})\right)^{\Delta}(t) + \int_a^b q(t)(1-p(\delta(t,\xi)))\left(\frac{\delta(t,b}{2}\right)(x^{\Delta}(\delta(t,\xi))\Delta\xi \leq 0,$$

$$\left(r(x^{\Delta})\right)^{\Delta}(t) + (b-a)q(t)(1-p(\delta(t,a)))\left(\frac{\delta(t,b)}{2}\right)(x^{\Delta}(\delta(t,a))) \leq 0,$$

or

$$\frac{\left(r(x^{\Delta})\right)^{\Delta}(t) + (b-a)q(t)(1-p(\delta(t,a)))}{r(\delta(t,a))},$$

$$\left(\frac{\delta(t,b)}{2}\right)(r(x^{\Delta})\delta(t,a)) \leq 0$$

which is the inequality (2).

As a consequence of this, we have a contradiction and therefore every solution of Equation (1) oscillates.

Theorem 2

Assume that (H_1) and (H_2) hold. In addition, assume that $r^{\Delta}(t) \geq 0$, $\delta(t.\xi)$ is increasing with respect to t and there exists a positive right dense continuous, Δ differentiable function $\alpha(t)$ such that

$$\lim_{t\to\infty} Sup \int_{t_0}^{t} \left(\alpha(s)Q(s) - \frac{((\alpha^{\Delta}(s))_{+})^2 r(\delta(s,b))}{4\alpha(s)} \right) = \infty \qquad (11)$$

where $(\alpha^{\Delta}(t))_{+} = \max\{\alpha^{\Delta}(t), 0\}$ and

$Q(s) = (b-a)q(s)p(s)(1-p(\delta(s,a)))$. Then every solution of Equation (1) is oscillatory (Higgins, 2008; Saker, 2010; Thandapani, *et al.*, 2011).

Proof

Let $y(t)$ be a non-oscillatory solution of (1).

Without loss of generality assume that $y(t) > 0$ for $t \geq t_0$, then

$y(\tau(t)) > 0$ and $y(\delta(t,\xi)) > 0$ for $t \geq t_1 > t_0$

and $b \geq \xi \geq a$.

Define the function $z(t) = \alpha(t)\dfrac{r(t)x^{\Delta}(t)}{x(\delta(t,b))}$, $\quad t \geq t_4$

Then $z(t) > 0$. Now

$$z^{\Delta}(t) = \alpha(t)\left(\frac{r(t)x^{\Delta}(t)}{x(\delta(t,b))}\right)^{\Delta} + \left(r(t)x^{\Delta}(t)\right)^{\sigma}\left(\frac{x(\delta(t,b))\alpha^{\Delta}(t) - \alpha(t)(x(\delta(t,b)))^{\Delta}}{x(\delta(t,b))(x^{\sigma}(\delta(t,b)))}\right)$$

$$\leq -\alpha(t)\,\mathrm{Q(t)} + \frac{\alpha^{\Delta}(t)z^{\sigma}(t)}{\alpha^{\sigma}(t)} - \frac{\alpha(t)}{(\alpha^{\sigma}(t))^2 r(\delta(t,b))}(z^{\sigma}(t))^2$$

$$\leq -\alpha(t)\,\mathrm{Q(t)} + \frac{\alpha^{\Delta}(t)z^{\sigma}(t)}{\alpha^{\sigma}(t)} - \frac{\alpha(t)(r(t)x^{\Delta}(t))^{\sigma}x(\delta(t,b))^{\Delta}}{x(\delta(t,b))x^{\sigma}(\delta(t,b))}.$$

$$\leq -\left(\alpha(t)Q(t) - \frac{(\alpha^{\Delta}(t))_{+}{}^2 r(\delta(t,b))}{4\alpha(t)}\right)$$

Integrating from t_7 to t, we obtain

$$z(t_7) \geq \int_{t_7}^{t} \left(\alpha(\mathrm{s})Q(\mathrm{s}) - \frac{(\alpha^{\Delta}(\mathrm{s}))_{+}{}^2 r(\delta(\mathrm{s},b))}{4\alpha(\mathrm{s})}\right)\Delta s,$$

which contradicts Equation (11)

Hence the proof.

Example: Consider the following second order neutral dynamic equation (Candan, 2011; Candan, 2013).

$$\left[\left(y(t) + \frac{t+a-1}{t+a}y(\tau(t))\right)^{\Delta}\right]^{\Delta} + \int_a^b t^{\frac{-1}{3}}\,\mathrm{y(t-}\xi)\Delta\xi = 0 \qquad (12)$$

Here $r(t) = 1$, $\mathrm{p}(t) = \dfrac{t+a-1}{t+a}$, $q(t) = t^{\frac{-1}{3}}$.

CONCLUSION

All the conditions of Theorem (2) are satisfied.

Now $Q(s) = (b-a)s^{\frac{-1}{3}}\left(1 - \dfrac{s-1}{s}\right)$, $= (b-a)s^{\frac{-1}{3}}\left(\dfrac{1}{s}\right)$

Taking $\alpha(s) = s$, we see that

$$\lim_{t\to\infty} Sup \int_{t_0}^{t} \left(\alpha(s)Q(s) - \frac{((\alpha^{\Delta}(s))_{+})^2 r(\delta(s,b))}{4\alpha(s)}\right)$$

$$= \lim_{t\to\infty} Sup \int_{t_0}^{t} \left((b-a)s^{\frac{-1}{3}} - \frac{1}{4s}\right)\Delta s = \infty$$

Therefore (12) is oscillatory.

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