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Research

REVISITING THE DEPENDENCE OF SOUND TRANSMISSION ON THE ANGLE OF INCIDENCE AT THE INTERFACE BETWEEN MEDIA OR MASSIVE LAYER

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ABSTRACT

Problems related to sound transmission through the interface between media and plates are the most popular in technical and architectural acoustics. The problems concerned normal incidence of sound have long been solved, while results obtained are confirmed by experiments and widely used in practice. Theoretical solutions to the problem of oblique sound incidence have not received direct experimental confirmation, and therefore a number of auxiliary theories were proposed during the last century allowing bringing together the results of calculation and experiment. In this article we consider the problem of the transmission of plane harmonic wave through a planar interface of a liquid or gaseous non- viscous media, as well as through the plates, showing a systemic error in the solutions to the problems of oblique transmission of sound. This error is caused by the fact that just the oscillation speeds of the incident, reflected, and transmitted waves were taken into account when carrying out calculations. The fact of variations in cross-sections of the sound rays in the incident and refracted waves depending on the incidence and refraction angles and, hence, variations in the ratios of the masses, encompassed in the oscillatory process, were not taken into account. The proposed physical model of the sound transmission takes into account in equal measure both factors: the vector of oscillatory speed and the variation in the cross section of the sound ray. The model is based on the momentum and kinetic energy conservation laws. The results of calculations according to the proposed model are well confirmed by practice.

INTRODUCTION

In acoustics and wave mechanics, the problem of an oblique sound incidence at the media interface is solved employing Snell's (Snellius) laws and the Fresnel equations obtained for light beams and well-proven by optical experiments. In acoustics, Fresnel equations for sound transmission and reflection coefficients are derived theoretically by solving continuity equations of particle velocity and sound pressure at the media interface, and consist of ratios of mechanical impedance of the media. The rays of the incident, reflected, and transmitted waves are represented by vectors in the form of line segments, deprived of the transverse dimension (Fig. 1), while the continuity condition is considered at the point of vectors intersection. In other words, the physical model of sound transmission excludes from consideration cross-sectional factor of the ray.

At striking an interface at normal incidence, cross-sections of sound rays of the incident, reflected, and transmitted waves are the same, while the vectors of the particle velocity of incident, reflected, and transmitted wave lie on the same line. The Fresnel formulas for sound transmission and reflection coefficients have the following form:

$$\alpha = \frac{2\rho_2 c_2}{\rho_2 c_2 + \rho_1 c_1}, \beta = \frac{\rho_2 c_2 - \rho_1 c_1}{\rho_2 c_2 + \rho_1 c} \quad (1)$$

where ρ is the density of the medium; c − is the

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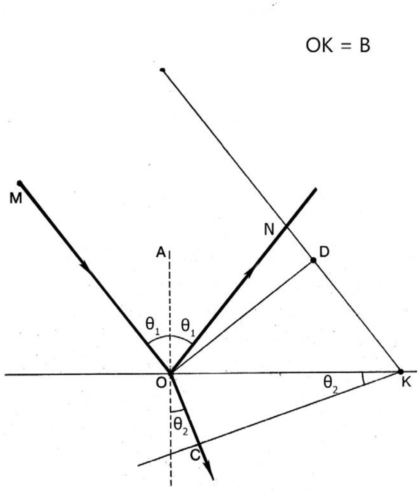

Fig. 1 Ratio between widths of the incident (OD) and refracted (KC) sound rays. The width of the contact spot of the incident and refracted rays is B = OK.

speed of sound.

Using the known relationship between speed of sound with a wavelength of **λ** and frequency **f**: c = λ · f, formulas (1) can be rewritten in the following form:

$$\alpha = \frac{2\rho_2\lambda_2}{\rho_2\lambda_2 + \rho_1\lambda_1}, \beta = \frac{\rho_2\lambda_2 - \rho_1\lambda_1}{\rho_2\lambda_2 + \rho_1\lambda_1} \quad (2)$$

The dimension of terms in formulas (2) is [kg/m^2]. We can imagine that the terms in the formulas represent the mass of media volume, equal to the product of the contact spot area of the incident and transmitted rays and the wavelengths propagating in the first and second media. Obviously, at the time of the wave incidence at the interface, the volume of a medium, encompassed by the incident wave, contained a certain amount of kinetic energy and momentum (the effective value) of the oscillating particles of the medium. At the time after the wave transmission, kinetic energy and momentum distributed between the volumes of the media encompassed by the reflected and transmitted waves. Masses of the considered volumes corresponding to these timepoints are well defined and represent a closed system (Khaikin, 1947) obeying the momentum and kinetic energy conservation laws.

Taking the cross-sectional area of the sound ray as a unit, and denoting the effective value of particle velocities of the medium by v, the momentum conservation equation can be written as:

$$\rho_1\lambda_1 v = \rho_1\lambda_1\alpha v + \rho_2\lambda_2\beta v \quad (3)$$

while conservation equation of kinetic energy:

$$\frac{\rho_1\lambda_1 v^2}{2} = \rho_1\lambda_1(\alpha v)^2/2 + \rho_2\lambda_2(\beta v)^2/2 \quad (4)$$

The solution to the system of equations (3) and (4) produces the following formulas for transmission and reflection coefficients:

$$\alpha = \frac{2\rho_1\lambda_1}{\rho_1\lambda_1 + \rho_2\lambda_2}, \beta = \frac{\rho_1\lambda_1 - \rho_2\lambda_2}{\rho_1\lambda_1 + \rho_2\lambda_2} \quad (5)$$

The comparison of obtained expressions with the Fresnel formulas (1) shows that at a constant overall structure of the formulas, order of the indexes denoting the media is changed. Comparison of results of calculation conducted by formulas (1) and (5) with experimental data (the propagation of sound through the interface between air and sea water) (Isakovich, 1973) showed that formulas (1) give the reflection and transmission coefficients based on sound pressure, while the formula (5) − by particle velocity (Zakharov, 2016).

At oblique angles of sound incidence, according to the Snell law, the angle of incidence θ_1 and the angle of refraction θ_2 at the interface of acoustically different media always are unequal, though according to the continuity condition, the traces of incident and refracted rays are always equal. This means that the cross sections of the rays OD and OK are not equal (Fig. 1).

In the modern literature (Yavorsky, *et al.*, 2006), the Fresnel formulas for the transmission and reflection coefficients at an oblique incidence of the wave have the following form:

$$\alpha = \frac{2\rho_2 c_2 \cos\theta_1}{\rho_2 c_2 \cos\theta_1 + \rho_1 c_1 \cos\theta_2}$$

$$\beta = \frac{\rho_2 c_2 \cos\theta_1 - \rho_1 c_1 \cos\theta_2}{\rho_2 c_2 \cos\theta_1 + \rho_1 c_1 \cos\theta_2}.$$

Cosines of the incidence and refraction angles represent directions of the vectors.

To obtain expressions for the transmission and reflection coefficients from the conservation equations, it is necessary to know masses of the media volumes encompassed by the rays of the incident and transmitted waves, and, consequently, their cross-sectional areas. We assume that the cross-sectional area of the ray varies depending on the angle of incidence only owing to the width of the ray lying in the plane of the drawing. Then they can be obtained by multiplying the width B of the contact spot of the incident and reflected waves on the cosines of the incidence and refraction angles of the sound rays. Thus, the mass of the volume of the medium encompassed by the ray within the length of the incident (and reflected) wave is equal to ρ_{11} B cos θ_1, while for transmitted wave it will be $\rho_2\lambda_2$ B$\cos\theta_2$. The particle velocity in the wave at incident angle of θ_1 to the normal of the interface will be v/cos θ_1, in reflected wave β/cos θ_1, and in refracted wave αv/cos θ_2. The momentum of the medium within a wavelength encompassed by incident ray equals to:

$$(\rho_1\lambda_1 Bcos\theta_1)\cdot\frac{v}{cos\ \theta_1}=\rho_1\lambda_1\beta v$$

by ray of the reflected wave - ${}_1\lambda_1$ Bβv, and by ray of the refracted wave - $\rho_2\lambda_2$ Bαv. Taking the width of the contact area B to be equal to unity at oblique sound incidence, we obtain same momentum conservation equation and the kinetic energy conservation equation as for normal incidence. From this it follows that the sound transmission and reflection coefficients do not depend on the angles of incidence, though this conclusion is contrary to popular belief.

When solving this task we used the technique, which is based on consideration of a certain volume separated out of the medium and containing mass characterizing this volume. This approach allowed developing a discrete model of the sound transmission based on use of the conservation equations of mechanics.

Let's use this technique when considering the problem of sound transmission through the plate (Fig. 2) separating a gaseous or liquid medium. The "mass action law", well known in architectural and engineering acoustics, describes airborne sound insulation by means of a massive plate:

$$R=10\lg\frac{1}{\alpha^2}\cong 20\lg\frac{\pi mf}{\rho c},dB \quad (16)$$

where ρ – is the density of the medium, separated by the plate, [kg·m^{-3}]; m – is the surface density of the plate [kg·m^{-2}]; c – is the sound speed in the medium [m·s^{-1}], f – is the oscillation frequency [s^{-1}].

In the physical model of formula (6), the value m is the concentrated mass, which is exposed to linear sound ray. Practice has shown that under the condition of normal incidence of sound on the plate, at which cross-sectional areas of rays of the incident and transmitted waves are equal to the landing area of the rays on the plate, and at frequencies below the limiting frequency of the wave coincidence, the formula provides the calculations, which are quite close to the actual values.

In the same way as in the previous problem, using the dependence of the wavelength on the sound speed and oscillation frequency, the "mass action law" of airborne sound insulation by means of a massive plate can be obtained from joint solution of the momentum conservation equation:

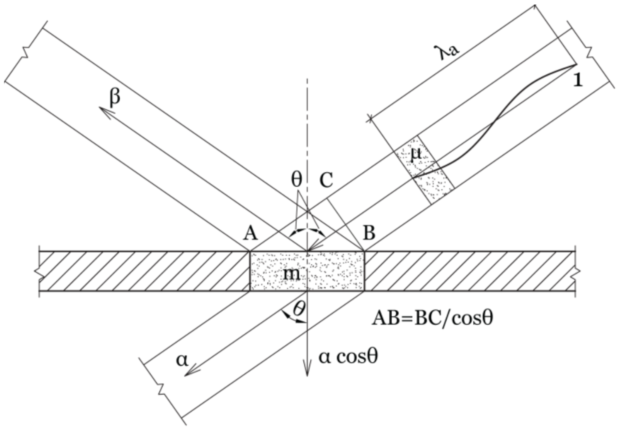

Fig. 2 The ratio between the rays' width and their contact spot on the plate.

$$\frac{\rho\lambda}{2\pi}v=\frac{\rho\lambda}{2\pi}av+(\frac{\rho\lambda}{2\pi}+m)\beta v \quad (7)$$

and kinetic energy conservation equation

$$\frac{\rho\lambda}{2\pi}v^2/2=\frac{\rho\lambda}{2\pi}(av)^2/2+(\frac{\rho\lambda}{2\pi}+\mathrm{m})(\beta v)^2\backslash 2 \quad (8)$$

Transmission coefficient of particle velocity is:

$$\alpha=\frac{2\rho\lambda}{2\rho\lambda+m},$$

Acoustic insulation can be calculated by the formula:

$$R=10\lg\frac{1}{\alpha^2}=20\lg(\frac{\pi m}{\rho\lambda}+1),\mathrm{dB},$$

For massive plates at $\mathrm{m}>>\rho\lambda, R=20\lg\frac{\pi m}{\rho\lambda},\mathrm{dB},$

When sound wave strikes an interface at angle θ to the normal of the plate (Fig. 3) the formula of the mass action law takes the following form (Zaborov, 1969):

$$\mathrm{R}=20\lg\frac{\pi m}{\rho\lambda}cos\ \theta,dB \quad (9)$$

It is obvious from (Fig. 2) that the width of the contact spot of sound rays AV is more than their cross sections BS by 1/cosθ times. Therefore, in equations (7) and (8) it is necessary to substitute m by m/cosθ. Then formula (9) will be converted back to formula (6). This implies that the sound insulation of the plate does not depend on the angle of incidence. This situation is observed within the range of frequencies below the limiting frequency of the wave coincidence. In this frequency range, flexural stiffness of the plate is small, and the external wave is opposed by the inertia of the plate, which is deformed to follow the shape of the impacted wave like a cloth trembling in the wind. It is known that in any body performing vibrations, elastic forces resist

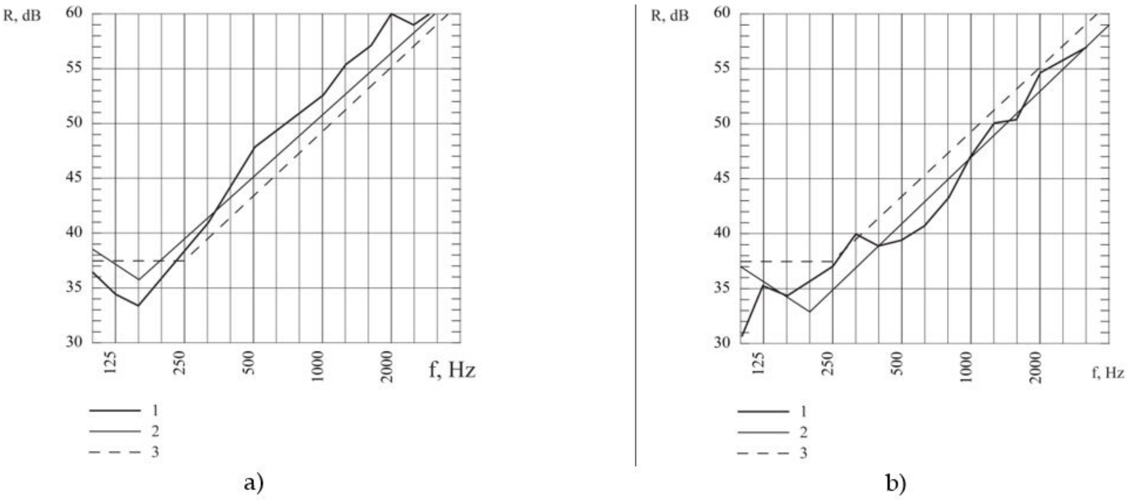

Fig. 3 Comparison of experimental curves 1 (Fasold and Sontag 1972) with the calculations of acoustic insulation by the proposed method (curves 2 (Zakharov 2012)) and normative method (curves 3 (SP 23-103-2003, 2004)): a) reinforced concrete wall 120 mm thick, b) gas-silicate wall 100 mm thick.

the forces of inertia. At wave coincidence, where a trace of the incident wave in the medium coincides with the length of the flexural wave in a plate, inertial resistance is weakened due to the elasticity effects, while transmission of sound enhances. Though, the phenomenon of wave coincidence is not a resonance (Zakharov, 2014). The wave coincidence occurs at the interfaces of any media due to their continuity property. The peculiarity of the wave coincidence at sound transmission through the plate is that within the considered frequency range, the plate is not inertial barrier, while it is the medium, in which transverse waves, and in particular, flexural waves propagate. Since the plate is incompressible along its thickness, the incident wave is transmitting simultaneously in two media: the plate and the media behind it.

To derive the formula for sound insulation by plate within the frequency range of the wave coincidence, we use equations (7) and (8) introducing the notation $\mu = \frac{\rho\lambda}{2\pi}$ and substituting m by $\mu_{fl} = m\,\lambda_{fl}/2\pi$. The first value indicates the reduced mass (Zakharov, 1973; Zakharov, 2012) of the air medium, in which sound waves propagate, while the second one indicates the reduced mass of the plate, where flexural waves propagate. The reduced mass is a concentrated mass, whose action on other bodies and the media is equivalent to the action of the medium, which is represented by this mass. The application of reduced mass is convenient in calculations based on use of discrete models.

In consequence of joint solution of equations (7) and (8), after changes introduced, the formula of sound insulation of plate within the frequency range of the sound coincidence will be as follows:

$$R = 20\lg\frac{\mu_{fl}}{2\mu} = 20\lg\frac{m}{2\rho\lambda} = 20\lg\frac{mf}{2\rho c},\ dB \qquad (10)$$

As is obvious, the obtained formula differs from the formula (6) by the value of 2π, corresponding to approximately 16 dB. Comparison of calculation results of insulation of the plate made of dense materials, carried out by this formula, with measured data shows the best convergence as compared with the results calculated by all other known formulas.

The insulation of the plate within the range below the limit frequency of wave coincidence is greatly influenced by interference of waves that have transmitted into the plate from the outside, and waves reflected from the plate edges (Zakharov, 1973). The reflection coefficient of the particle velocity from the edges largely depends on the hardness and stiffness of the test plate mounting to the adjacent structures, whereas experimentalists, as becomes clear from their descriptions of test conditions, pay attention just to the tightness of contiguity of the plate along the contour. As a result, in most cases, data allowing taking into account the effect of sound reflection coefficients on acoustic insulation, are lacking. Statistical results of the field measurements in the buildings made of stone and concrete showed a deviation of sound insulation from the values, calculated according to the formula (6), for walls and ceilings within the frequency range around 100 Hz − up to 9 dB, through two octaves or at the limit frequencies of the wave coincidence − up to 3 dB. Thus, for practical calculations within the frequency range below the limit frequency of the wave coincidence, it is expedient to determine the acoustic insulation by the formula (6) with subsequent reduction of the calculated values by 6 dB, while within the range above the limit frequency of the wave coincidence − by the formula (10).

As an example, (Fig. 3) shows the experimental and calculated insulation spectra of walls made of heavy concrete and gas silicate. The solid line shows a calculated spectrum obtained by the proposed method, dotted line corresponds to calculations by a standard method (SP 23-103-2003, 2004), while bold line represents experimental data (Fasold and Sontag, 1972). As is obvious, the calculated lines corresponding to the proposed method are located from different sides of the line, calculated by a standard method, though in both cases they are closer to the experimental curves. This indicates the fact that the proposed method provides calculation accuracy, which is higher than the specified accuracy.

The discrete models to calculate the acoustic insulation of single-ply protection walls are described in detail in (Zakharov, 2012), which presents also the comparison analysis of calculation results corresponding to normative methods, with experimental and field data (in total 46 third-octave spectra), published in domestic and foreign periodicals. Samples made of dense materials (stone, concrete, glass, and plastics) with thickness of 300−3 mm and surface density of 500−3 kg/m^2 were studied. The average deviations of calculations by the proposed method with regard to the experimental data were 2.4 dB, whereas the deviations for the normative methods were 2.7 dB.

CONCLUSIONS

1. The work shows that sound transmission through the interface and the sound insulation of plate at frequencies below the limiting frequency of the wave coincidence do not depend on the angle of incidence.

2. Demonstrated calculation method of sound insulation, based on discrete physical model of sound transmission, enables applying equations of conservation laws of mechanics. The proposed method is simple, while the calculation results are in good agreement with experimental and practical data.

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