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Research Article

SOME CHARACTERIZATION OF COMPLEMENTARY TRIPLE CONNECTED DOMINATION NUMBER OF 3 – REGULAR GRAPHS

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ABSTRACT

A graph is said to be triple connected if any three vertices lie on a path in G. A subset S of V of a nontrivial graph G is said to be complementary triple connected dominating set, if S is a dominating set and the induced sub graph <V-S> is triple connected. The minimum cardinality taken over all complementary triple connected dominating sets is called the complementary triple connected domination number of G and is denoted by Υctc (G). The minimum number of colours required to colour all the vertices such that adjacent vertices do not receive the same colour is called chromatic number $\chi(G)$. In this paper, we investigate 3 - regular graphs for which Υctc (G)= X(G)=3.

INTRODUCTION

The concept of triple connected graphs was introduced by (Paulraj, *et al.*, 2012). A graph is said to be triple connected if any three vertices lie on a path in G. In (Paulraj, *et al.*, 2012) the authors introduced triple connected domination number of a graph. A subset S of V of a nontrivial graph G is said to be triple connected dominating set is S is a dominating set and <S> is a triple connected. The minimum cardinality taken over all triple connected dominating sets is called the triple connected domination number of G and is denoted by $\gamma_{tc}(G)$.

The concept of complementary triple connected graphs was introduced by (Harary, 1972; Haynes, *et al.*, 1998; Mahadevan, *et al.*, 2013). A subset S of V of a nontrivial graph G is said to be complementary triple connected dominating set, if S is a dominating set and the induced sub graph <V-S> is triple connected. The minimum cardinality taken over all complementary triple connected dominating sets is called the complementary triple connected domination number of G and is denoted by $\gamma_{ctc}(G)$.

The minimum number of colours required to colour all the vertices such that adjacent vertices do not receive the same colour is called chromatic number $\chi(G)$ Motivated by the above concept we investigate 3 - regular graphs whose complementary triple connected domination number equals chromatic number equals three. We use the following results.

MAIN RESULTS

Observation

For any connected graph G, $\gamma_{ctc}(G) \le n-3$

Let G = (V,E) be a connected 3 - regular graph of order p with $\gamma(G) \le \gamma_{ctc}(G)$ and $\left\lceil \frac{p}{\Delta+1} \right\rceil \le \gamma_{ctc} \le p-3$. We consider 3 - regular graph for which $\chi(G) = \gamma_{ctc}(G) = 3$. $\left\lceil \frac{p}{\Delta+1} \right\rceil \le \gamma_{ctc}$ which gives $\left\lceil \frac{p}{4} \right\rceil \le 3$, p ≤ 12. Since $\gamma_{ctc}(G) = 3$, 6 ≤ p ≤ 12 and since G is 3 - regular, p must be even. Hence p = 6 or 8 or 10 or 12

Theorem

There does not exist a connected 3 - regular graph on 6 vertices, whose chromatic number equal to

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complementary triple connected domination number equals 3 (Mahadevan, *et al*., 2013).

Proof

Let V-S = {x,y,z}. Since $<V-S>$ is triple connected, $\langle S\rangle \neq \overline{K}_3$, $K_2 \cup K_1$. Hence $<V-S> = K_3$ or P_3. Let $<V-S> = K_3$ or P_3. Then the possible case of $<S> = K_3$, P_3. In all the above cases it is not possible to form 3 - regular graph, which is a contradiction. Hence no new graph exists.

Theorem

Let G be a connected 3 - regular graph on 8 vertices. Then $\chi(G) = \gamma_{ctc}(G) = 3$, if and only if G is isomorphic to J_1 (Fig. 1)

Proof

Let G be a connected 3 - regular graph on 8 vertices with $\chi(G) = \gamma_{ctc}(G) = 3$. Since $\gamma_{ctc}(G)$ = 3, there exists a ctc- set with 3 elements. Let S = {x,y,z} be such a set. Let V−S = { v_1, v_2, v_3, v_4, v_5}. $<S> = K_3$ or $\overline{K_3}$ or $K_1 \cup K_2$ or P_3 and $<V-S> = \overline{K_5}$, P_5, $P_4 \cup K_1$, $P_3 \cup \overline{K}_2$, $P_2 \cup \overline{K}_3$, $P_2 \cup P_2 \cup K_1$, $P_3 \cup K_2$, $K_{1,4}$, $K_{1,3} \cup K_1$, C_5, $K_4 \cup K_1$, $C_4(P_2)$, K_5, W_5, F_2,, K_5-{e}, $K_4 \cup K_1$, $K_4(P_2)$, $K_3 \cup \overline{K}_2$, $C_3(P_2) \cup K_1$, $K_3 \cup K_2$, $C_3(P_3)$, $C_3(2P_2)$, $C_3(P_2, P_2, 0)$ and the following (Fig. 2).

Since G is 3 - regular, and S is a γ_{ctc} - set the only possible graph of $< V-S> = P_5$ or C_5

Case 1: Let $< V-S> = C_5 = (v_1, v_2, v_3, v_4, v_5)$

Since G is 3 - regular, it is clearly verified that $<S> \neq K_3$ or $\overline{K_3}$ or $K_1 \cup K_2$. Hence the only possible if $<S> = P_3 = (x, y, z)$. Since G is connected 3 - regular, y must be adjacent to any one of the vertex of C_5. Let y be adjacent to v_1, since G is 3 - regular, x is adjacent to any two of { v_2, v_5} or v_2 (or v_5) and v_3(or v_4). Let x be adjacent to v_2 and v_5. Then z is must be adjacent to v_3 and v_4, so that $G \cong H_1$. If x is adjacent to {v_2, v_3}, then z is must be adjacent to v_4 and v_5. In this stage {x, v_2} is γ_{ctc} - set, which is a contradiction. Hence no graph exists (Sivagnanam, 2012).

Case 2: $< V-S> = P_5 = (v_1, v_2, v_3, v_4, v_5)$

Since G is 3 - regular, it is clearly verified that $<S> \neq K_3$ or $\overline{K_3}$. Hence $<S> = K_2 \cup K_1$ or P_3.

Sub case (i) Let $<S> = K_2 \cup K_1$

Let xy be the edge in $K_2 \cup K_1$. Since G is connected let z be adjacent to both v_1 and v_5 and any one of {v_2, v_3, v_4} or z is adjacent to v_1(or v_5) and any two of {v_2, v_3, v_4} or z is adjacent to v_2 and v_3 and v_4. Let z is adjacent to v_1,v_5 and v_2. Then x is adjacent to v_1 and v_5 or x is adjacent to v_1 and v_3(or v_4) or x is adjacent to v_3 and v_4 or x is adjacent to v_5 and any one of {v_3, v_4}. If x is adjacent to v_1 and v_5, then y must be adjacent to v_3 and v_4. In this stage {v_1, v_4} be a γ_{ctc} - set, which is a contradiction. Hence no graph exists. If x is adjacent to v_1 and v_3 and then y must be adjacent to v_4 and v_5. In this stage {v_1, v_4} be a γ_{ctc} - set, which is a contradiction. Hence no graph exists. If x is adjacent to v_3 and v_4, then y must be adjacent to v_5 and v_1. In this stage {v_1, v_4} be a γ_{ctc} - set, which is a contradiction. Hence no graph exists. If x is adjacent to v_5 and v_3, then y must be adjacent to v_4 and v_1 so that $G \cong H_1$.

Let z be adjacent to v_1, v_2, v_3. Then x is adjacent to v_4 and v_5 or v_1 and v_4 or v_1 and v_5. If x is adjacent to v_4 and v_5, then y must be adjacent to v_1 and v_5. In this stage {v_5, z} be a γ_{ctc} - set, which is a contradiction. Hence no graph exists. If x is adjacent to v_1 and v_4, then y must be adjacent to v_5, since G is 3 - regular, which is a contradiction. If x is adjacent to v_1 and v_5, then y must be adjacent to v_4 and v_5. In this stage {y, z} be a γ_{ctc} - set, which is a contradiction. Hence no graph exists. Let z be adjacent to v_2, $v_3$and v_4. Since G is 3 - regular, x be adjacent to v_1 and v_5, which is a contradiction.

Sub case (ii) Let $<S> = P_3(x, y, z)$

Since G is 3 - regular, y is adjacent to v_1 (or v_5) or any one of {v_2, v_3, v_4}. If y is adjacent to v_1, since G is 3 - regular, x is cannot be v_1 and v_5 and also x cannot be adjacent to v_1 and any one {v_2, v_3, v_4} and x cannot be adjacent to any two of {v_2, v_3, v_4}, which is a contradiction. If y is adjacent to v_3, then x cannot be adjacent to v_1 and v_3, and also x cannot be adjacent to v_2 and v_3, which is a contradiction. Hence no graph exists.

Three - Regular Graphs on 10 Vertices

Let G be a connected 3 - regular graph on 10 vertices for which $\chi(G) = \gamma_{ctc}(G) = 3$. Let s = {x,y,z} be a complementary triple connected dominating set. Since G is 3 - regular, clearly $<S> \neq K_3$ and P_3. Hence $<S> = K_1 \cup K_2$ or $\overline{K_3}$. Let $<S_1> = N(x) = \{v_1, v_2, v_3\}$. Let $<S> = K_1 \cup K_2$

Let yz be the edge in $<S>$. Let v_4 and v_5 be the vertices adjacent to y and v_6 and v_7 be adjacent to z. Let $<S_2> = \{ v_4, v_5\}$ and $<S_3> = \{ v_6, v_7\}$. Then we will consider following three cases.

(i) $<S_1> = P_3$

$<S_1> = \overline{K_3}$

$<S_1> = K_2 \cup K_1$

Proposition: 3. 1

If $<S> = K_1 \cup K_2$ and $<S_1> = P_3$, then G is isomorphic to ς_j for $1 \le j \le 3$ (Fig. 3).

Proof:

Let <S_1> = P_3 ={v_1, v_2, v_3}. We consider the following three cases.

Case1 <S_2> = <S_3> = K_2

Case2 <S_2> = K_2 and <S_3> = $\overline{K_2}$

Case3 <S_2> =<S_3> = $\overline{K_2}$

Case 1: <S_2> = <S_3> = K_2

Since G is 3 - regular, v_1 is adjacent to any one of { v_4, v_5, v_6, v_7}. Let v_1 be adjacent to v_4. Since G is 3 - regular, v_3 cannot be adjacent to v_5. Hence v_3 is adjacent to any one of { v_6, v_7}. Let v_3 be adjacent to v_6. Hence v_5 must be adjacent to v_7 so that $G \cong G_1$

Case 2: <S_2> = K_2 and <S_3> = $\overline{K_2}$

If v_7 is adjacent to v_4 and v_5 or v_1 and v_3 or v_1 (or v_3) and v_4(or v_5). If v_7 is adjacent to v_4 and v_5, then v_6 is adjacent to v_1 and v_3 so that <V-S> is not triple connected, which is a contradiction. If v_7 is adjacent to v_1 and v_3, then v_6 is adjacent to v_4 and v_5 so that <V-S> is not triple connected, which is a contradiction. Hence v_7 is adjacent to v_1 and v_4, then v_6 is adjacent to v_3 and v_5 so that $G \cong G_2$

Case 3: <S_2> =<S_3> = $G \cong G_3$

Since G is 3 - regular, v_1 is adjacent to any one of { v_4, v_5, v_6, v_7}. Let v_1 is adjacent to v_4. Since G is 3 - regular, v_4 cannot be adjacent to v_3. Hence v_4 is adjacent to v_6(or v_7) and then v_7 is adjacent to v_3 and v_5 and since G is 3 - regular, v_5 is adjacent to v_6 so that $G \cong G_3$.

Proposition: 3.2

If <S> = $K_1 \cup K_2$ and <S_1> = ς_j , then G is isomorphic to ς_j for $4 \le j \le 7$ (Fig. 4).

Proof: We consider the following three cases.

Case 1: <S_2> = <S_3> = K_2

<V-S> has seven vertices for which for which three vertices are of deg 1 and the remaining 4 are of degree 2. Within this is not possible to form a 3 - regular graph, Hence no graph exists.

Case 2: <S_2> = K_2 and <S_3> = $\overline{K_2}$

v_1 is adjacent to v_4 and v_5 (or) v_6 and v_7 (or) v_4 (or v_5) and v_6(or v_7). If v_1 is adjacent to v_4 and v_5, then v_2 is adjacent to v_6 and v_7 and then v_3 is adjacent to v_6 and v_7. In this stage if we take S = { v_4, v_2, v_7} with v_2v_7 ЄE(G), then S is a dominating set. Let S_1= {v_5, y, v_1}. Now, in this situation <S> = $K_1 \cup K_2$, <S_1> = P_3 which falls under proposition:3.1

If v_1 is adjacent to v_6 and v_7, then v_2 is adjacent to v_4 and v_5,(or) v_6 and v_7 (or) v_4(or v_5) and v_6(or v_7). If v_2 is adjacent to v_4 andv_5, then v_3 is adjacent to v_6 and v_7. In this stage if we take S = { v_5,v_1,v_6} with v_1v_6 ЄE(G), then S is a dominating set. Let S_1= {y, v_4, v_2}. Now, in this situation <S> = $K_1 \cup K_2$, <S_1> = P_3 which falls under proposition:2.1. If v_2 is adjacent to v_4 andv_6, then v_3 is adjacent to v_7 and v_5 so that $G \cong G_4$.

v_1 is adjacent to v_4 and v_6. Since G is 3 - regular, v_2 is not adjacent to v_5 and v_6 and hence it must be adjacent to v_5 and v_7(or) v_6 and v_7. If v_2 is adjacent to v_5 and v_7, then v_3 is adjacent to v_6 and v_7 so that $G \cong G_4$.

Case 3: <S_2> =<S_3> = $\overline{K_2}$

Now v_1 is adjacent to v_4 and v_5(or v_6 and v_7) or v_4 and v_6. If v_1 is adjacent to v_4 and v_5, then since G is 3 - regular, v_2 is not adjacent to v_4 and v_5. Hence v_2 is adjacent to v_6 and v_7 or v_4 (or v_3) and v_6(or v_7).

If v_2 is adjacent to v_6 and v_7, then v_3 is not adjacent to v_4 and v_7. Also v_3 is not adjacent v_6 and v_7. Hence v_3 is adjacent to v_4(or v_5) and v_6 (or v_7) and then v_7 is adjacent to v_5 so that $G \cong G_5$. If v_2 is adjacent v_4 and v_6, then v_7 is adjacent to v_5 and v_3 and then v_3 is adjacent to v_6 so that $G \cong G_6$.

If v_1 is adjacent to v_6 and v_4, then v_2 is adjacent to v_6 and v_4 or v_7 and v_5 or v_6 and v_7(or v_4 and v_5) or v_6 and v_7 (or v_7 and v_4). If v_2 is adjacent to v_6 and v_4,then v_3 is adjacent v_7 and v_5 and then v_5 is adjacent to v_7 so that <V-S> is not triple connected, which is a contradiction.

If v_2 is adjacent to v_7 and v_5, then v_3 cannot be adjacent to v_4 and v_7 (or v_6 and v_7). Hence v_3 is adjacent to v_4 and v_6 or v_5 and v_7 or v_4 and v_7 (or v_6 and v_7).

If v_3 is adjacent is adjacent to v_4 and v_6, then v_7 is adjacent to v_5 so that <V-S> is not triple connected. If v_3 is adjacent to v_5 and v_7, then v_6 is adjacent to v_4 so that <V-S> is not triple connected. If v_3 is adjacent to v_7 and v_4, then v_6 is adjacent to v_5 so that $G \cong G_6$.

If v_2 is adjacent to v_6 and v_5, then v_3 cannot be adjacent to v_4 and v_5. Hence v_3 is adjacent to v_4 and v_7 or v_7 and v_5. If v_3 is adjacent to v_7 and v_4, then v_7 is adjacent to v_5 so that $G \cong G_7$. If v_3 is adjacent to v_5 and v_7, then v_7 is adjacent to v_4 so that $G \cong G_7$.

If v_2 is adjacent to v_6 and v_7, then v_3 cannot be adjacent to v_7 and v_4. Also v_3 is not adjacent to v_7 and v_5. Hence v_3 is adjacent to v_4 and v_5 and then v_5 is adjacent to v_7 so that $G \cong G_5$.

Proposition: 3.3

If <S> = $K_1 \cup K_2$ and <S_1> = $K_2 \cup K_1$, then G is isomorphic to ς_j for $8 \le j \le 11$ (Fig. 5).

Proof: Let $v_1 v_2$ be the edge in <S$_1$>. We consider the following three cases.

Case 1: $<S_2> = <S_3> = K_2$

Now, v_3 is adjacent to v_4 and v_5 (or v_6 and v_7) or v_6 and v_4 (or v_7 and v_5). If v_3 is adjacent to v_4 and v_5, then v_2 is adjacent t to v_6 (or v_7) and then v_1 is adjacent to v_7. In this stage, if we take S = { v_4, v_2,v_6} with $v_2v_6 \in E(G)$, then S is a dominating set. Let $S_1 = (v_5,y,v_3\}$. Now in this situation $<S>=K_1\cup K_2$, $<S_1>= P_3$ which follows under proposition 3.1 If v_3 is adjacent to v_6 and v_4, then v_1 is adjacent v_7 (or v_5) and then v_2 is adjacent to v_5 so that $G \cong G_8$.

Case 2: $<S_2> = K_2$ and $<S_3> = \overline{K_2}$

Since G is 3 - regular, v_3 cannot be adjacent to v_4 and v_5. Hence v_3 is adjacent to v_6 (or v_7) and v_4(or v_5) and v_6 (or v_5). If v_3 is adjacent to v_6 and v_7, then v_6 is adjacent to v_1(or v_2) or v_4(or v_5). If v_6 is adjacent to v_1, then v_7 is not adjacent to v_2 and hence v_7 is adjacent to v_4(or v_5) and then v_5 is adjacent to v_2 so that $G \cong G_9$.

If v_6 is adjacent to v_4, then v_7 is not adjacent to v_5. Hence v_7 is adjacent to v_1(or v_2) and then v_2 is adjacent to v_5 so that $G \cong G_9$. If v_3 is adjacent to v_4 and v_6, then v_7 is adjacent to v_1 and v_2 or v_1(or v_2) and v_5. If v_7 is adjacent to v_1 and v_2, then v_6 is adjacent to v_5. In this stage, if we take S = { v_5, v_6,v_1} with $v_5v_6 \in E(G)$, then S is a dominating set. Let $S_1 = (x,v_2,v_7\}$. Now in this situation $<S> = K_1 \cup K_2$, $<S_1> = P_3$ which follows under proposition 3.1. If v_7 is adjacent to v_1 and v_5, then v_2 is adjacent to v_6 so that $G \cong G_{10}$.

Case 3: $<S_2> =<S_3> = \overline{K_2}$

Now v_3 is adjacent to v_6 and v_7(or v_4 and v_5) or v_6 and v_4(or v_7 and v_5). If v_3 is adjacent to v_6 and v_7. In this stage, if we take S = {x, v_3,y} with $xv_3 \in E(G)$, then S is a dominating set. Let $S_1 = (z,v_4,v_5\}$. Now in this situation $<S> = K_1\cup K_2$, $<S_1> = G \cong G_{11}$ which follows under proposition 3.2. If v_3 is adjacent to v_6 and v_4, then v_7 is not adjacent to v_1 and v_2. Hence v_7 is adjacent to v_4 and v_5 or v_1 and v_4 or v_1 and v_5.

If v_7 is adjacent to v_4 and v_5, then v_5 is not adjacent to v_6. Hence v_5 is adjacent to v_1(or v_2) and then v_2 is adjacent to v_6. In this stage, if we take S = {v_3,v_6,v_5} with $v_3v_6 \in E(G)$, then S is a dominating set. Let $S_1 = (y,v_7,v_1\}$. Now in this situation $<S> = K_1\cup K_2$, $<S_1> = G \cong G_{11}$ which follows under proposition 3.2. If v_7 is adjacent to v_1 and v_4, then v_5 is adjacent to v_2 and v_6 so that $G \cong G_{11}$.

If v_7 is adjacent to v_1 and v_5, then v_2 is not adjacent to v_6. Hence v_2 is adjacent to v_4 and v_5. If v_2 is adjacent to v_4, then v_5 is adjacent to v_6. In this stage, if we take S = {v_7,v_1,v_4} with $v_7v_1 \in E(G)$, then S is a dominating set. Let $S_1 = (y,v_3,v_6\}$, $S_2 = (v_5,z\}$, $S_3 = (v_2,x\}$. Now in this situation $<S> = K_1\cup K_2$, $<S_1> = K_2\cup K_1$, $<S_2> = \overline{K_2}$ and $<S_3> = K_2$ which falls under case 2.

Now we consider the graphs with $<S> = \overline{K_3}$. Then y can be adjacent to two of three vertices not in N[x]. If it is adjacent to two vertices say v_4 and v_5. Let $S_2 = \{v_4,v_5\}$. Then z is adjacent to two vertices say v_6 and v_7. Let $S_3 = \{v_6,v_7\}$. Then we will consider the following three situations.

(i) $<S_1> = P_3$

(ii) $<S> = K_2\cup K_1$

$<S_1> = \overline{K_3}$

Proposition 3. 4

If $<S> = \overline{K_3}$ and $<S_1> = P_3$, then no new graph exists.

Proof: Let $<S_1> = P_3 = (v_1v_2v_3)$. We consider the following 3 cases.

Case 1: $<S_2> = <S_3> = K_2$

Let $U = S_2\cup\{y\}$ and $V = S_3\cup\{z\}$. Then $<U> = <V> = C_3$. Since G is 3 - regular, for some $u \in U$ and $v \in V$, $uv \in E(G)$. Then for S = {x,u,v} is a dominating set. Now in this situation $<S> = K_1\cup K_2$, which falls under propositions $<S> = K_1\cup K_2$.

Case 2: $<S_2> = K_2$ and $<S_3> = \overline{K_2}$

Let v_4 and v_5 be the edge in S_2. Since G is 3 - regular y cannot be adjacent to v_1(or v_3). Hence y is adjacent to v_6 (or v_7). If y is adjacent to v_6(or v_7), then v_7 is adjacent v_1 and v_3 or v_5 and v_4 or v_4(or v_5) and v_1(or v_3). If v_7 is adjacent to v_1 and v_3, then v_6 is adjacent to v_4(or v_5) and then z is adjacent to v_5. In this situation if we take S = {x,v_5,z} with $v_5z \in E(G)$ then S is a dominating set Let $S_1 = \{v_1,v_2,v_3\}$. Now in this situation $<S> = K_1 \cup K_2$, $<S_1> = P_3$ which falls under proposition 3.1. If v_7 is adjacent v_1 and v_4, then v_5 is not adjacent v_3. Hence v_5 is adjacent to v_6 or z. If v_5 is adjacent to v_6, then z is adjacent to v_3 so that <V¯S> is not triple connected, which is a contradiction. If v_5 is adjacent to z, then v_3 is adjacent to v_6. In this stage, if we take S = { x, v_5,z} with $v_5z \in E(G)$, then S is a dominating set. Let $S_1 = (v_1,v_2,v_3\}$. Now in this situation $<S> = K_1\cup K_2$, $<S_1> = P_3$ which follows under proposition 3.1. If v_7 is adjacent to v_5 and v_4, then v_6 is adjacent to v_1(or v_3) and z is adjacent to v_3 so that $<V-S>$ is not triple connected, which is a contradiction.

Case 3: $<S_2> =<S_3> = \overline{K_2}$

Now y is adjacent to v_1(or v_3) or v6(or v_7). In both cases no new graph exists.

Proposition 3. 5

If $<S> = \varsigma_j$ and $<S_1> = K_2\cup yK_1$, then G is isomorphic to ς_j for $11 \le j \le 13$ (Fig. 6).

Proof: Let v_1v_2 be the edge in $<S_1>$. We considers the following three cases.

Case 1: $<S_2> = <S_3> = K_2$

Now v_1 is adjacent to any one of $\{v_4,v_5,y\}$(or any one of$\{v_6,v_7,w\}$). Let v_1 is adjacent to v_4. Then no new graph exists.

Case 2: $<S_2> = K_2$ and $<S_3> = \overline{K_2}$

Now v_1 is adjacent to any one of $\{y,v_4,v_5\}$ or z or v_6 (or v_7). If v_1 is adjacent to y, then z is adjacent to v_3 or v_4(or v_5) or v_2. Let z is adjacent to v_3. In this stage, if we take $S = \{ z,v_1,y\}$ with $v_1y \in E(G)$, then S is a dominating set. Let $S_1 = (v_3,v_6,v_7\}$. Now in this situation $<S> = K_1 \cup K_2$, $<S_1> = \overline{K_3}$ which follows under proposition 3.2. If z is adjacent to v_4. In this stage, if we take $S = \{ x,v_4,z\}$ with $v_4z \in E(G)$, then S is a dominating set. Let $S_1 = (v_1,v_2,v_3\}$. Now in this situation $<S> = K_1 \cup K_2$, $<S_1> = K_1 \cup K_2$ which follows under proposition: 3.3. If z is adjacent to v_2, then v_5 is adjacent to v_3 or v_6(or v_7). Let v_5 is adjacent to v_3. In this stage, if we take $S = \{ v_5,v_2,z\}$ with $v_2z \in E(G)$, then S is a dominating set. Let $S_1 = (v_3,v_4,y\}$. Now in this situation $<S> = K_1 \cup K_2$, $<S_1> = K_1 \cup K_2$ which follows under proposition 3.3. If v_5 is adjacent to v_6, then v_7 is adjacent to v_1 and v_4, and then v_3 is adjacent to v_6 so that $G \cong G_{12}$. If v_1 is adjacent to z, then no new graph exists. If v_1 is adjacent to v_6, then v_6 is adjacent to any one of$\{y,v_4,v_5\}$ or v_3 or v_2. If v_6 is adjacent to y, then z is adjacent to v_3 or v_2 or v_4 (or v_5). If z is adjacent to v_3, then v_2 is adjacent to v_4 (or v_5) or v_7. If v_2 is adjacent to v_4, then v_7 is adjacent to v_3 and v_5. In this stage, if we take $S = \{ y,v_3,x\}$ with $v_3x \in E(G)$, then S is a dominating set. Let $S_1 = (v_4,v_5,v_6\}$. Now in this situation $<S> = K_1 \cup K_2$, $<S_1> = K_1 \cup K_2$ which follows under proposition 3.3. If v_2 is adjacent to v_7, then since G is 3 - regular v_7 cannot be adjacent to v_3. Hence v_7 must be adjacent to v_4 (or v_5) and then v_3 is adjacent to v_5 so that $G \cong G_{13}$. If z is adjacent to v_2 or v_4, then no new graph exists. If v_6 is adjacent to v_3 or v_2, then no new graph exists.

Case 3: $<S_2> = <S_3> = \overline{K_2}$

Now v_1 be adjacent to y(or z) or v_4(or v_5){or v_6(or v_7)}. In both cases no new graph exists.

Proposition: 3.6

If $<S> = \overline{K_3}$ and $<S_1> = \overline{K_3}$, then G is isomorphic to G_{14} (Fig. 7).

Proof: We consider the following three cases.

Case 1: $<S_2> = <S_3> = K_2$

Let $v_4 v_5$ be the edge in $<S_2>$ and v_6v_7 be the edge in $<S_3>$. Now v_1 is adjacent to any two of $\{y,v_4,v_5\}$ (or equivalently $\{ z,v_6,v_7\}$) or any one of $\{y,v_4,v_5\}$ and any one of$\{z,v_6,v_7\}$. In both cases no new graph exists.

Case 2: $<S_2> = K_2$ and $<S_3> = \overline{K_2}$

Now z is adjacent to any one of $\{v_1,v_2,v_3\}$. Let z be adjacent to v_1. Then y is adjacent to v_1 or not adjacent to v_1. In both cases no new graph exists.

Case 3: $<S_2> = <S_3> = \overline{K_3}$

Now v_1 is adjacent to v_4 (or v_5) or v_1 is adjacent to y (or z). If v_1 is adjacent to v_4, then y is adjacent to v_1 or v_2 (or v_7). If y is adjacent to v_1. In this stage, if we take $S = \{ y,x,z\}$, then S is a dominating set. Let $S_1 = (v_1,v_4,v_5\}$. Now in this situation $<S> = \overline{K_3}$, $<S> = K_1 \cup K_2$ which follows under proposition 3.5. If y is adjacent to v_2, then z is adjacent to v_2 or v_1(or v_4) or v_3 (or v_5). If z is adjacent v_2, then v_6 is adjacent to v_3 and v_5 or v_1 and v_4 or v_3 (or v_5) and v_1(or v_4). If v_6 is adjacent to v_3 and v_5, then v_4 is adjacent to v_3 or v_7. If v_4 is adjacent to v_3, then v_7 is adjacent to v_1 and v_5 so that $<V-S>$ is not triple connected, which is a contradiction.

If v_4 is adjacent to v_7, then v_1 is adjacent to v_5 or v_7, then v_1 is adjacent to v_5 or v_7. If v_1 is adjacent to v_5, then v_3 is adjacent to v_7 so that$<V-S>$ is not triple connected, which is a contradiction. If v_1 is adjacent to v_7, then v_3 is adjacent to v_5. In this stage, if we take $S = \{ v_3,v_2,v_4\}$, then S is a dominating set. Let $S_1 = (x,v_5,v_6\}$. Now in this situation $<S> = \overline{K_3}$, $<S_1> = K_1 \cup K_2$ which follows under proposition 3.5. If v_6 is adjacent to v_1 and v_4, then v_5 is adjacent to v_3 and v_7 and then v_3 is adjacent to v_7. In this stage, if we take $S = \{ v_2,v_4,v_3\}$, then S is a dominating set. Let $S_1 = (y,v_1,v_6\}$. Now in this situation $<S> = \overline{K_3}$, $<S_1> = K_1 \cup K_2$ which follows under proposition 3.5. If v_6 is adjacent to v_3 and v_1, then v_5 is adjacent to v_3 and v_7, and then v_7 is adjacent to v_4 so that $<V-S>$ is not triple connected, which is a contradiction. If z is adjacent to v_1 or v_3, then no new graph exists. If y is adjacent to v_6, then z is adjacent to v_1 or v_4 or v_5 or v_2(or v3). If z is adjacent to v_1, then no new graph exists. If z is adjacent to v_4, then v_6 is adjacent to v_1 or v_2(or v_3). If v6 is adjacent to v_1, then v_2 is adjacent to v_5 and v_7 and then v_3 is adjacent to v_5 and v_7 so that $<V-S>$ is not triple connected, which is a contradiction.

If v_6 is adjacent to v_2, then v_1 is adjacent to v_5 or v_7. If v_1 is adjacent to v_5, then v_3 is adjacent to v_5 and v_7, and then v_2 is adjacent to v_7 so that $G \cong G_{14}$. If v1 is adjacent to v_7, then v_3 is adjacent to v_5 and v_7, and then v_2 is adjacent to v_5 so that $G \cong G_{14}$. If z is adjacent to v_5 or v_2, then no new graph exists. If v_1 is adjacent to y, then no new graph exists.

Theorem 3.7

Let G be a 3 - regular graph of order 10. Then $\chi(G) = \gamma_{ctc}(G) = 3$, if and only if G is isomorphic to graphs ς_j, 1 ≤ j ≤ 14

Proof

If G is any one of the graphs in figures in ς_j, then clearly $\chi(G) = \gamma_{ctc}(G) = 3$. Conversely, assume that $\chi(G) = \gamma_{ctc}(G) = 3$. Then the proof follows from proposition 3.1 to 3.6

CONCLUSION

In this paper, we characterized the 3- regular graphs for which $\chi(G) = \gamma_{ctc}(G) = 3$ of order up to 10 vertices. The authors are also characterized 3-regular graphs for which $\chi(G) = \gamma_{ctc}(G) = 3$ of order 12 vertices which will be report in the subsequent papers.

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