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Research Article

TENSOR PAIRS AND DOMAIN PAIRS OF CESIUM DIHYDROGEN PHOSPHATE USING GROUP THEORETICAL TECHNIQUES

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ABSTRACT

Cesium Dihydrogen Phosphate (CsH2PO4) solid single crystal, it shows different phase transitions Cubic structure is changed to Rhombohedra structure and Cubic to monoclinic structure with various temperatures. In this paper, Physical property tensor pairs and domain pairs of CsH2PO4 at different phase transitions are calculated.

INTRODUCTION

The crystal CsH_2PO_4 (read as CDP: Cesium Dihydrogen Phosphate) under phase transition exhibit pseudo cubic direction rhombohederal, monclinic or tetragonal and this exhibit "gaint peizo – electric coefficients" and high electromechanical coupling factors.

Due to the phase transitions of the crystal $Cs_2H_2PO_4$ (Cesium Dihydrogen Phosphate) cubic structure (m3m (Oh Group) Prototypic point group) is changed into Rhombohedra structure (2 (C_2 group) ferroic point group) is around 154.2K temperature. Again, this crystal changed this structure into Rhombohedra structure (space group p2/m (C_{2h} group) ferroic point group) is around upto ap. 505K With different temperatures. Because of its peizo electric property this crystal does not exhibit any magnetic property. At $P2_1$ it exhibits ferroelectric, ferro elastic and magneto electric polarizibility (MEP) discussed.

Cesium Dihydrogen phosphate undergoes a number of unusual properties which have so far not been found in the other ferroelectric systems. The dielectric constant of CDP shows pronounced deviation from the Curie-Weiss law over a large temperature range above θ_c (Aizu, 1973; Aizu, 1974; Badurski and Stroz, 1979). Also, anomalously large excess heat capacities remain over the same temperature range (Abusahmin, 2017). A detailed Raman spectroscopy investigations of CDP revealed the existence of the antiferro electric fluctuations in the vicinity of the paraelectric-ferroelectric phase transition (Bhagavantam and Pantulu, 1964;). Furthermore, it has been found from pyroelectric investigations that the para electric phase exhibits fluctuations of polarization, and the pyroelectric charge does not disappear at θ_c but persist on heating up to ap. 230 K decreasing non-monotonically (Bradley and Cracknell, 1972; Jaffe, *et al*., 1971; Narayana, *et al*., 1990).

A ferroic crystal contains one or more domains but of the different spatial orientation, a ferroic crystal

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arises in a ferroic phase transition from phase of higher symmetry to a phase of lower symmetry, here grey groups $G1^1$ is the prototypic point group and H is a ferroic phase of a lower symmetry. (Aizu, 1970) has given all possible 773 species of the ferroic crystals in phase transitions (Radha, 2013; Karri, *et al.*, 2009; Karri, 2011).

The domains in ferroic crystals can be switched by means of a magnetic field , an electric field, a mechanical stress or a combination of the and consequently ferroic crystals of technological importance for memory storage and electric and magnetic switches.

The domain pairs and tensor pairs of Ferro electric, Ferro elastic and Magneto electric polarizability for the ferro species by using coset &double coset decomposition taking grey group $m3m1^1$ as the prototypic point group in case of magneto electric polarizability and in case of ferro electric & ferro elastic polarizibility for ordinary point group m3m were calculated and tabulated.

REPRESENTATIVE DOMAIN PAIRS

The group G and the subgroup F one writes the left coset decomposition of G with respect to F symbolically as $G = F + g_1F + g_2F + \ldots + g_nF$

Where g_iF, I = 1, 2, 3, … n denotes the subset of elements of G, which is obtained by multiplying each element of the subgroup F from the left by the elements g_i of G. Each subset of elements of g_iF, i = 1, 2, 3, …n of G are called left coset representatives of the left coset decomposition of G with respect to F (Janovec, 1989).

The subset of elements of G in each coset of the left coset decomposition of G with respect to F is unique but the coset representatives are not unique. A coset representative g_i can be replaced by the element g_if where f is an arbitrary element of a sub group F. $S_i = g_iS_1$ i.e., the orientation of the i^{th} domain S_i is related to the orientation of the domain S_1 by the element g_i of this coset decomposition for i = 1, 2, … ,n the symmetry group $F_i = g_i F g_i^{-1}$ i.e., the groups F and F_i are conjugate groups. Two domain states S_i and S_j form a domain pair (S_i , S_j) if $S_j = g_{ij}S_j$ and $S_i = g_{ij}S_i$ where g_{ij} is element of G (Karri and Babovic, 2017; Litvin and Litvin, 1990; Uma and Sireesha, 2013).

REPRESENTATIVE TENSOR PAIRS

Let G be the prototypic point group, H is the ferroic point group and T is the specific form of the physical property tensor T that keeps H invariant. The number N of crystallographically equivalent ordered distinct tensor pair classes is equal to the number of double cosets decomposition of G with respect to G_T.

$G = G_TEG_T + G_T g_1G_T + \ldots + G_T g_N G_T$

Where G_T is the stabilizer of T in G and g_k k = 1, 2, …n are the double coset representatives.

Let "T" denote a spontaneous physical property tensor which arises in the low symmetry phase of the crystal. Denote by $T^{(i),}$ i = 1, 2, … q, the specific form of the tensor T characterizing each of the q domains, and denote $T^{(1)} = T$. All ordered tensor pairs could be partitioned into classes of crystallography equivalent tensor pairs : Two tensor pairs $(T^{(i)}, T^{(j)})$ and $(T^{(i^{`})}, T^{(j^{`})})$ are said to be crystallographically equivalent with respect to G and to belong to the same class of ordered tensor pairs, if there is an element g of G such that $(T^{(i)}, T^{(j)}) = (gT^{(i)}, gT^{(j)})$ that is , if $T^{(i)} = gT^{(i^{`})}$ and $T^{(j)} = gT^{(j^{`})}$.

Let G_T denote the stabilizer of T in G, this subgroup G_T of G is the set of all elements g of G which leave invariant i.e., gT = T. if G_T = H then T is a full physical property tensor and there are $q_T = q$ distinct forms of tensor T i.e., each of the q domains is characterized by a distinct form of the tensor T. if H is a subgroup of G_T then T is a partial physical property tensor and there are $q_T \leq q$ distinct forms of the tensor T by $T^a_{(d)}$, a = 1, 2, … q_T and and choose $T_d^{(1)} = T^{(1)} = T$.

RESULTS AND DISCUSSION

All ordered distinct tensor pairs $(T_d^{(a)}, T_d^{(b)})$ can be partitioned into classes of crystallographically equivalent ordered distinct tensor pairs in the same manner as $T^{(i)}, T^{(j)}$). The number of classes of ordered distinct tensor pairs is same as the number of classes of tensor pairs (Litvin amd Wike, 1989).

(i) Ferro–electric domain pairs for CsH_2PO_4 in the state m3m F 2:

Consider the ferroic species m3m F 2, where m3m is a prototypic point group and 2 is a ferroic point group. The number of distinct domain pair classes is 12. The coset decomposition of m3m with respect to the group '2' is given by

$$G = m3m = E(2) + C_{2a}(2) + C_{2c}(2) + C_{2d}(2) + C_{2e}(2) + C_{2f}(2) + C_{2x}(2) + I(2) + C_{31}^{+}(2) + C_{32}^{+}(2) + C_{31}^{-}(2) + C_{4z}^{+}(2) + C_{4z}^{+}(2) + \sigma_x(2) + S_{61}^{+}(2) + S_{62}^{+}(2) + S_{4z}^{+}(2) + S_{61}^{-}(2) + S_{62}^{-}(2) + \sigma_{da}(2) + \sigma_{dc}(2) + \sigma_{dd}(2) + \sigma_{de}(2) + \sigma_{df}(2)$$

The coset elements g_i`s are E, C_{2a}, $C_{2c,}$ C_{2d}, C_{2e}, C_{2f}, I, $C_{31}^{+}, C_{32}^{+}, C_{31}^{-}, C_{32}^{-}, C_{4z}^{+}, S_{61}^{+}, S_{62}^{+}, S_{61}^{-}, S_{62}^{-}$,

σ_{da} , σ_{dc}, σ_{dd} , $\sigma_{de}, \sigma_{df}, S_{4z}^{+}$. Now let $S_i = C_{2a}$, $g_{ij} = S_{4z}^{+}$ and $S_j = S_{4z}^{+}$, then we have $S_i = g_{ij} S_j$ i.e., $C_{2a} = \sigma_x S_{4z}^{+}$ and $S_{4z}^{+} = \sigma_x C_{2a}$. hence (C_{2a}, S_{4z}^{+}) forms a domain pair, instead of writing this we represent domain pair representatives of G = m3m are (E, σ_x), (C_{2x}, I), (C_{2a}, S_{4z}^{+}), (C_{2c}, σ_{de}), (C_{2d}, σ_{df}), (C_{2e}, σ_{dc}), (C_{31}^{-}, S_{62}^{+}), (C_{31}^{-}, S_{62}^{+}), (C_{32}^{-}, S_{61}^{+}), (C_{31}^{+}, S_{62}^{-}), (C_{31}^{+}, S_{62}^{-}) and $(\sigma_{da}, C_{4z}^{+})$. The domain pairs for ferroic species m3m F 2 are tabulated in Table 1.

(ii) Ferro-electric tensor pairs For CsH_2PO_4 in the state m3m F 2:

Consider the ferroic species m3m F 2 where m3m is a prototypic point group and 2 is a ferroic point group and the stabilizer G_T is 4mm. The numbers of distinct tensor pair classes are 3.

The double coset decomposition of m3m with respect to the stabilizer 4 mm is given by

G = m3m = (4 mm) E (4 mm) + (4 mm) C_{2x} (4 mm) + (4 mm) C_{31}^{+} (4 mm)

Here m3m is a prototypic point group and stabilizer 4 mm is a ferroic point group. The ferro-electro tensor pairs for ferroic species m3m F 2 are tabulated in Table 2.

Table 1. Ferroelectric domain pairs for ferroic species m3m F2

Domain pair representatives	Domain Pairs	
(E, σ_x)	(0, 0, Z)	(0, 0, Z)
(C_{2x}, I)	(0, 0, -Z)	(0, 0, -Z)
(C_{2a}, S_{4z}^{+})	(0, 0, -Z)	(0, 0, -Z)
(C_{2c}, σ_{de})	(Z, 0, 0)	(Z, 0, 0)
(C_{2d}, σ_{df})	(0, Z, 0)	(0, Z, 0)
(C_{2e}, σ_{dc})	(-Z, 0, 0)	(-Z, 0, 0)
(C_{2f}, σ_{dd})	(0, -Z, 0)	(0, -Z, 0)
(C_{31}^{-}, S_{62}^{+})	(0, Z, 0)	(0, Z, 0)
(C_{32}^{-}, S_{61}^{+})	(0, -Z, 0)	(0, -Z, 0)
(C_{31}^{+}, S_{62}^{-})	(Z, 0, 0)	(Z, 0, 0)
(C_{32}^{+}, S_{61}^{-})	(-Z, 0, 0)	(-Z, 0, 0)
$(\sigma_{da}, C_{4z}^{+})$	(0, 0, Z)	(0, 0, -Z)

Table 2. Ferro-electro tensor pairs for ferroic species m3m F2

Double coset representatives	Stabilizer G_T	Tensor Pairs	
(a)	(b)	(c)	(d)
E	4 mm	(0, 0, Z)	(0, 0, Z)
C_{2x}	4 mm	(0, 0, Z)	(0, 0, Z)
C_3^{+}	4 mm	(0, 0, Z)	(Z, 0, 0)

(iii) Ferro-Elastic domain pairs for CsH_2PO_4 in the state m3m F 2:

Consider the ferroic species m3m F 2 where m3m is a prototypic point group and 2 is a ferroic point group. The number of distinct domain pair classes is 12. The coset decomposition of m3m with respect to the group 2 is given by

$$G = m3m = E(2) + C_{2a}(2) + C_{2c}(2) + C_{2d}(2) + C_{2e}(2) + C_{2f}(2) + C_{2x}(2) + I(2) + C_{31}^{+}(2) + C_{32}^{+}(2) + C_{31}^{-}(2) + C_{32}^{-}(2) + C_{4z}^{+}(2) + \sigma_x(2) + S_{61}^{+}(2) + S_{62}^{+}(2) + S_{4z}^{+}(2) + S_{61}^{-}(2) + S_{62}^{-}(2) + \sigma_{da}(2) + \sigma_{dc}(2) + \sigma_{dd}(2) + \sigma_{de}(2) + \sigma_{df}(2)$$

The coset elements g_i`s are $E, C_{2a}, C_{2c}, C_{2d}, C_{2e}, C_{2f}, I, C_{2x}, C_{31}^{+}, C_{32}^{+}, C_{31}^{-}, C_{32}^{-}, C_{4z}^{+}, S_{61}^{+}, S_{62}^{+}, S_{61}^{-}, S_{62}^{-}, \sigma_{da}, \sigma_{dc}, \sigma_{dd}, \sigma_{de}, \sigma_{df}, S_{4z}^{+}$.

The Ferro Elastic Domain pairs representatives of m3m F 2 are

(E, σ_x), (C_{2x}, I), (C_{2a}, S_{4z}^{+}), (C_{2c}, σ_{de}), (C_{2e}, σ_{dc}), (C_{2e}, σ_{dc}), (C_{2f}, σ_{dd}), (C_{31}^{-}, S_{62}^{+}), (C_{32}^{+}, S_{61}^{-}), (C_{32}^{+}, S_{61}^{-}), (C_{31}^{+}, S_{62}^{-}) & $(\sigma_{da}, C_{4z}^{+})$.
The Ferro Elastic Domain pairs of m3m F 2 are tabulated in Table 3.

(iv) Ferro-Elastic Tensor pairs for CsH_2PO_4 in the state m3m F 2:

Consider the ferroic species m3m F 2 where m3m is a prototypic point group and 2 is a ferroic point group and the stabilizer G_T is 2/m. The number of distinct tensor pair classes is 8. The double coset decomposition of m3m with respect to the stabilizer 2/m is given by

G = m3m = (2/m) E (2/m) + (2/m) C_{2a} (2/m) + (2/m) C_{2c} (2/m) + (2/m) C_{2d} (2/m) + (2/m) C_{31}^{-} (2/m) + (2/m) C_{2x} (2/m) + (2/m) C_3^{-} (2/m) + (2/m) C_{31}^{+} (2/m) . The Ferro Elastic tensor pairs of m3m F 2 are tabulated in Table 4.

Table 3. Ferro-elastic domain pairs for ferroic species m3m F 2

Domain Pair Representatives	Domain pairs	
(E, σ_x)	(xx, yy, zz, xy)	(xx, yy,zz, $\bar{x}$y)
(C_{2x}, I)	(xx, yy,zz,x$\bar{y}$)	(xx, yy, zz, xy)
(C_{2a}, S_{4z}^{+})	(yy, xx, zz, yx)	(yy, xx, zz, $\bar{y}$x)
(C_{2c}, σ_{de})	(zz, yy, xx, zy)	(zz, yy, xx, zy)
(C_{2d}, σ_{df})	(xx, zz, yy, $\bar{x}$z)	(xx, zz, yy, xz)
(C_{2e}, σ_{dc})	(zz, yy, xx,$\bar{y}$ $\bar{y}$)	(zz, yy, xx, $\bar{z}$y)
(C_{2f}, σ_{dd})	(xx, zz, yy, $\bar{x}$ $\bar{z}$)	(xx, zz, yy, x$\bar{z}$)
(C_{31}^{-}, S_{62}^{+})	(yy, zz, xx, yz)	(yy, zz, xx, $\bar{y}$z)
(C_{32}^{-}, S_{61}^{+})	(yy, zz, xx, y$\bar{z}$)	(yy, zz, xx, $\bar{z}$ $\bar{z}$)
(C_{31}^{+}, S_{62}^{-})	(yy, zz, xx, $\bar{y}$z)	(yy, zz, xx, $\bar{y}$z)
(C_{32}^{+}, S_{61}^{-})	(zz, xx, yy, $\bar{z}$x)	(zz, xx, yy, $\bar{x}$ $\bar{x}$)
$(\sigma_{da}, C_{4z}^{+})$	(yy, xx, zz,$\bar{x}$ $\bar{x}$)	(yy, xx, zz, y$\bar{x}$)

Table 4. Ferro-elastic tensor pairs for ferroic species m3m F2

Double Coset Representations	Stabilizer	Tensor Pairs	
(a)	G_T	(b)	(c)
E	2/m	$\begin{pmatrix} xx & xy & 0 \\ 0 & yy & 0 \\ 0 & 0 & zz \end{pmatrix}$	$\begin{pmatrix} xx & xy & 0 \\ 0 & yy & 0 \\ 0 & 0 & zz \end{pmatrix}$
C_{2a}	2/m	$\begin{pmatrix} xx & xy & 0 \\ 0 & yy & 0 \\ 0 & 0 & zz \end{pmatrix}$	$\begin{pmatrix} yy & yx & 0 \\ 0 & xx & 0 \\ 0 & 0 & zz \end{pmatrix}$
C_{2c}	2/m	$\begin{pmatrix} xx & xy & 0 \\ 0 & yy & 0 \\ 0 & 0 & zz \end{pmatrix}$	$\begin{pmatrix} zz & z\bar{y} & 0 \\ 0 & yy & 0 \\ 0 & 0 & xx \end{pmatrix}$
C_{2d}	2/m	$\begin{pmatrix} xx & xy & 0 \\ 0 & yy & 0 \\ 0 & 0 & zz \end{pmatrix}$	$\begin{pmatrix} xx & \bar{x}z & 0 \\ 0 & zz & 0 \\ 0 & 0 & yy \end{pmatrix}$
C_{4z}^{-}	2/m	$\begin{pmatrix} xx & xy & 0 \\ 0 & yy & 0 \\ 0 & 0 & zz \end{pmatrix}$	$\begin{pmatrix} yy & y\bar{x} & 0 \\ 0 & xx & 0 \\ 0 & 0 & zz \end{pmatrix}$
C_{2x}	2/m	$\begin{pmatrix} xx & xy & 0 \\ 0 & yy & 0 \\ 0 & 0 & zz \end{pmatrix}$	$\begin{pmatrix} xx & x\bar{y} & 0 \\ 0 & yy & 0 \\ 0 & 0 & zz \end{pmatrix}$
C_{31}^{-}	2/m	$\begin{pmatrix} xx & xy & 0 \\ 0 & yy & 0 \\ 0 & 0 & zz \end{pmatrix}$	$\begin{pmatrix} yy & yz & 0 \\ 0 & zz & 0 \\ 0 & 0 & xx \end{pmatrix}$
C_{31}^{+}	2/m	$\begin{pmatrix} xx & xy & 0 \\ 0 & yy & 0 \\ 0 & 0 & zz \end{pmatrix}$	$\begin{pmatrix} zz & zx & 0 \\ 0 & xx & 0 \\ 0 & 0 & yy \end{pmatrix}$

(v) The Magneto-Electric polarizability (MEP) tensor pairs for CsH_2PO_4 in the state of $m3m1^l$ F 2:

Consider the ferroic species $m3m1^l$ F 2, where m3m is a prototypic point group and 2 is a ferroic point group and the stabilizer G_T is $2/m^l$. The number of distinct tensor pair classes is 22. The double coset decomposition of m3m with respect to the stabilizer $2/m^l$ is given by

G = $m3m1^l$ = $(2/m^l)$ E $(2/m^l)$ + $(2/m^l)$ R_2 $(2/m^l)$ + $(2/m^l)$ C_{2x} $(2/m^l)$ + $(2/m^l)$ C_{31}^{+} $(2/m^l)$ + $(2/m^l)$ C_{31}^{-} $(2/m^l)$ + $(2/m^l)$ C_{31}^{-} $(2/m^l)$ + $(2/m^l)$ C_{32}^{-} $(2/m^l)$ + $(2/m^l)$ C_{2a} $(2/m^l)$ + $(2/m^l)$ C_{2c} $(2/m^l)$ + $(2/m^l)$ C_{2d} $(2/m^l)$ + $(2/m^l)$ C_{2e} $(2/m^l)$ + $(2/m^l)$ C_{2f} $(2/m^l)$ + $(2/m^l)$ R_2 C_{2x} $(2/m^l)$ + $(2/m^l)$ $R_2 C_{31}^{+}$ $(2/m^l)$ + $(2/m^l)$

$R_2 C_{32}^{+}$ $(2/m^l)$ + $(2/m^l)$ $R_2 C_{31}^{-}$ $(2/m^l)$ + $(2/m^l)$ $R_2 C_{32}^{-}$ $(2/m^l)$ + $(2/m^l)$ $R_2 C_{2a}$ $(2/m^l)$ + $(2/m^l)$ $R_2 C_{2c}$ $(2/m^l)$ + $(2/m^l)$ $R_2 C_{2d}$ $(2/m^l)$ + $(2/m^l)$ $R_2 C_{2e}$ $(2/m^l)$ + $(2/m^l)$ $R_2 C_{2f}$ $(2/m^l)$

Table 5. The MEP tensor pairs for ferroic species $m3m1^l$F2

Double cosset representatives	Stabilizer (G_T)	Tensor Pairs	
(a)	(b)	(c)	(d)
E	$2/m^l$	$\begin{pmatrix} xx' & xy' & 0 \\ yx' & yy' & 0 \\ 0 & 0 & zz' \end{pmatrix}$	$\begin{pmatrix} xx' & xy' & 0 \\ yx' & yy' & 0 \\ 0 & 0 & zz' \end{pmatrix}$
R_2	$2/m^l$	$\begin{pmatrix} xx' & xy' & 0 \\ yx' & yy' & 0 \\ 0 & 0 & zz' \end{pmatrix}$	$\begin{pmatrix} -xx' & -xy' & 0 \\ -yx' & -yy' & 0 \\ 0 & 0 & -zz' \end{pmatrix}$
C_{2x}	$2/m^l$	$\begin{pmatrix} xx' & xy' & 0 \\ yx' & yy' & 0 \\ 0 & 0 & zz' \end{pmatrix}$	$\begin{pmatrix} xx' & -xy' & 0 \\ -yx' & yy' & 0 \\ 0 & 0 & zz' \end{pmatrix}$
C_{31}^{+}	$2/m^l$	$\begin{pmatrix} xx' & xy' & 0 \\ yx' & yy' & 0 \\ 0 & 0 & zz' \end{pmatrix}$	$\begin{pmatrix} zz' & 0 & 0 \\ 0 & xx' & xy' \\ 0 & yx' & yy' \end{pmatrix}$
C_{32}^{+}	$2/m^l$	$\begin{pmatrix} xx' & xy' & 0 \\ yx' & yy' & 0 \\ 0 & 0 & zz' \end{pmatrix}$	$\begin{pmatrix} zz' & 0 & 0 \\ 0 & xx' & -xy' \\ 0 & -yx' & yy' \end{pmatrix}$
C_{31}^{-}	$2/m^l$	$\begin{pmatrix} xx' & xy' & 0 \\ yx' & yy' & 0 \\ 0 & 0 & zz' \end{pmatrix}$	$\begin{pmatrix} yy' & 0 & yx' \\ 0 & zz' & 0 \\ xy' & 0 & xx' \end{pmatrix}$
C_{32}^{-}	$2/m^l$	$\begin{pmatrix} xx' & xy' & 0 \\ yx' & yy' & 0 \\ 0 & 0 & zz' \end{pmatrix}$	$\begin{pmatrix} yy' & 0 & -yx' \\ 0 & zz' & 0 \\ -xy' & 0 & xx' \end{pmatrix}$
C_{2a}	$2/m^l$	$\begin{pmatrix} yy' & yx' & 0 \\ xy' & xx' & 0 \\ 0 & 0 & zz' \end{pmatrix}$	$\begin{pmatrix} yy' & yx' & 0 \\ xy' & xx' & 0 \\ 0 & 0 & zz' \end{pmatrix}$
C_{2c}	$2/m^l$	$\begin{pmatrix} zz' & 0 & 0 \\ 0 & yy' & -yx' \\ 0 & -xy' & xx' \end{pmatrix}$	$\begin{pmatrix} zz' & 0 & 0 \\ 0 & yy' & -yx' \\ 0 & -xy' & xx' \end{pmatrix}$

The Magneto-Electric polarizability (MEP) tensor pairs of m3m1[1] F 2 are tabulated in Table 5.

(vi) Ferro-elastic domain pairs for CsH_2PO_4 in the state m3m F 2/m:

Consider the ferroic species m3m F 2/m where m3m is a prototypic point group and 2/m is a ferroic point group. The number of distinct domain pair classes is 6. The coset decomposition of m3m with respect to the group 2 is given by

C_{2d}	$2/m^1$	$\begin{pmatrix} xx' & xy' & 0 \\ yx' & yy' & 0 \\ 0 & 0 & zz' \end{pmatrix}$	$\begin{pmatrix} xx' & 0 & xy' \\ 0 & zz' & 0 \\ yx' & 0 & yy' \end{pmatrix}$
C_{2e}	$2/m^1$	$\begin{pmatrix} xx' & xy' & 0 \\ yx' & yy' & 0 \\ 0 & 0 & zz' \end{pmatrix}$	$\begin{pmatrix} zz' & 0 & 0 \\ 0 & yy' & yx' \\ 0 & xy' & xx' \end{pmatrix}$
C_{2f}	$2/m^1$	$\begin{pmatrix} xx' & xy' & 0 \\ yx' & yy' & 0 \\ 0 & 0 & zz' \end{pmatrix}$	$\begin{pmatrix} xx' & 0 & xy' \\ 0 & zz' & 0 \\ yx' & 0 & yy' \end{pmatrix}$
R_2C_{2x}	$2/m^1$	$\begin{pmatrix} -xx' & xy' & 0 \\ yx' & -yy' & 0 \\ 0 & 0 & -zz' \end{pmatrix}$	$\begin{pmatrix} -xx' & xy' & 0 \\ yx' & -yy' & 0 \\ 0 & 0 & -zz' \end{pmatrix}$
$R_2C_{31}^{+}$	$2/m^1$	$\begin{pmatrix} -zz' & 0 & 0 \\ 0 & -xx' & -xy' \\ 0 & -yx' & -yy' \end{pmatrix}$	$\begin{pmatrix} -zz' & 0 & 0 \\ 0 & -xx' & -xy' \\ 0 & -yx' & -yy' \end{pmatrix}$
$R_2C_{32}^{+}$	$2/m^1$	$\begin{pmatrix} xx' & xy' & 0 \\ yx' & yy' & 0 \\ 0 & 0 & zz' \end{pmatrix}$	$\begin{pmatrix} -zz' & 0 & 0 \\ 0 & -xx' & xy' \\ 0 & yx' & -yy' \end{pmatrix}$
$R_2C_{31}^{-}$	$2/m^1$	$\begin{pmatrix} xx' & xy' & 0 \\ yx' & yy' & 0 \\ 0 & 0 & zz' \end{pmatrix}$	$\begin{pmatrix} yy' & 0 & -yx' \\ 0 & -zz' & 0 \\ -xy' & 0 & -xx' \end{pmatrix}$
$R_2C_{32}^{-}$	$2/m^1$	$\begin{pmatrix} xx' & xy' & 0 \\ yx' & yy' & 0 \\ 0 & 0 & zz' \end{pmatrix}$	$\begin{pmatrix} -yy' & 0 & yx' \\ 0 & -zz' & 0 \\ xy' & 0 & -xx' \end{pmatrix}$
R_2C_{2a}	$2/m^1$	$\begin{pmatrix} xx' & xy' & 0 \\ yx' & yy' & 0 \\ 0 & 0 & zz' \end{pmatrix}$	$\begin{pmatrix} -yy' & -yx' & 0 \\ -xy' & -xx' & 0 \\ 0 & 0 & -zz' \end{pmatrix}$
R_2C_{2c}	$2/m^1$	$\begin{pmatrix} xx' & xy' & 0 \\ yx' & yy' & 0 \\ 0 & 0 & zz' \end{pmatrix}$	$\begin{pmatrix} -zz' & 0 & 0 \\ 0 & -yy' & yx' \\ 0 & xy' & -xx' \end{pmatrix}$
R_2C_{2d}	$2/m^1$	$\begin{pmatrix} xx' & xy' & 0 \\ yx' & yy' & 0 \\ 0 & 0 & zz' \end{pmatrix}$	$\begin{pmatrix} -xx' & 0 & -xy' \\ 0 & -zz' & 0 \\ -yx' & 0 & -yy' \end{pmatrix}$
R_2C_{2e}	$2/m^1$	$\begin{pmatrix} xx' & xy' & 0 \\ yx' & yy' & 0 \\ 0 & 0 & zz' \end{pmatrix}$	$\begin{pmatrix} -zz' & 0 & 0 \\ 0 & -yy' & -yx' \\ 0 & -xy' & -xx' \end{pmatrix}$
R_2C_{2f}	$2/m^1$	$\begin{pmatrix} xx' & xy' & 0 \\ yx' & yy' & 0 \\ 0 & 0 & zz' \end{pmatrix}$	$\begin{pmatrix} -xx' & 0 & -xy' \\ 0 & -zz' & 0 \\ -yx' & 0 & -yy' \end{pmatrix}$

Table 6. Ferro-elastic domain pairs for ferroic species m3m F 2

Domain Pair Representatives	Domain pairs	
(E, C_{2x})	(xx, yy, zz, xy)	(xx, yy,zz,x $\bar{y}$)
(C_{2a}, C_{4z}^{-})	(yy, xx, zz, yx)	(yy, xx, zz, y $\bar{x}$)
(C_{2c}, C_{2e})	(zz, yy, xx, z $\bar{y}$)	(zz, yy, xx, $\bar{y}$ $\bar{y}$)
(C_{2d}, C_{2f})	(xx, zz, yy, $\bar{x}$ z)	(xx, zz, yy, $\bar{z}$ $\bar{z}$)
(C_{31}^{+}, C_{32}^{+})	(zz, xx, yy, zx)	(zz, xx, yy, $\bar{z}$ x)
(C_{31}^{-}, C_{32}^{-})	(yy, zz, xx, yz)	(yy, zz, xx, y $\bar{z}$)

$$G = m3m = E(2/m) + C_{2a}(2/m) + C_{2c}(2/m) + C_{2d}(2/m) + C_{2e}(2/m) + C_{2f}(2/m) + C_{2x}(2/m) + C_{31}^{+}(2/m) + C_{32}^{+}(2/m) + C_{31}^{-}(2/m) + C_{32}^{-}(2/m) + C_{4z}^{-}(2/m)$$

The coset element g_i`s are $E, C_{2a}, C_{2c}, C_{2d}, C_{2e}, C_{2f}, C_{2x}, C_{31}^{+}, C_{32}^{+}, C_{31}^{-}, C_{32}^{-}, C_{4z}^{+}$

The Ferro Elastic Domain pair representatives of m3m F 2/m are (E, C_{2x}), (C_{2a}, C_{4z}^{-}), (C_{2c}, C_{2e}), (C_{2d}, C_{2f}), (C_{31}^{+}, C_{32}^{+}), (C_{31}^{-}, C_{32}^{-}) and The Ferro Elastic Domain pairs of m3m F 2/m are tabulated in Table 6.

(vii) Ferro-Elastic tensor pairs for CsH_2PO_4 in the state m3m F 2/m:

Consider the ferroic species m3m F 2/m where m3m is a prototypic point group and 2 is a ferroic point group and the stabilizer G_T is also same ferroic point group 2/m. The number of distinct tensor pair classes is 8. The double coset decomposition of m3m with respect to the stabilizer 2/m is given by

$$G = m3m = (2/m)\,E\,(2/m) + (2/m)\,C_{2a}\,(2/m) + (2/m)\,C_{2c}\,(2/m) + (2/m)\,C_{2d}\,(2/m) + (2/m)\,C_{4z}^{-}\,(2/m) + (2/m)\,C_{2x}\,(2/m) + (2/m)\,C_{31}^{-}\,(2/m) + (2/m)\,C_{31}^{+}\,(2/m)$$

CONCLUSION

In this paper the ferroelectric, ferroelastic, magneto electric polarizibility of the crystal CDP (Cesium Dihydrozen Phosphate with structural formula CsH_2PO_4) domain pairs & tensor pairs are calculated by group theoretical techniques. While considering ferroelectric and ferro elastic properties only ordinary point group P-m3m is considered as prototypic point group and 2, 2/m are ferroic point subgroups. Since they are non-magnetic properties. Ferroelectric and ferroelastic tensor pairs of m3m F2 is calculated by using the stabilizer 2 m, but in the case of magneto-electric polarizibility (MEP) grey group m3m1[1] is taken as prototypic point group and 2/m[l] is taken as stabilizer. Similarly Ferroelastic tensor pairs of m3m F2/m is calculated by same stabilizer 2/m[l].

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